CONTENTS

CHAPTER 1

Introduction

1.1 Definition

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

CONTENTS

1.2 Classifications of Machine Design

The machine design may be classified as follows :

1. ***Adaptive design.*** In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. ***Development design.*** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. ***New design.*** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows :

(*a*) ***Rational design.*** This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) ***Empirical design.*** This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

(*e*) ***System design.*** It is the design of any complex mechanical system like a motor car.

(*f*) ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(*g*) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

1.3 General Considerations in Machine Design

Following are the general considerations in designing a machine component :

1. ***Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed in chapters 4 and 5.

2. ***Motion of the parts or kinematics of the machine.*** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required. The motion of the parts may be :

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions.

(*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(*c*) Constant velocity.

(*d*) Constant or variable acceleration.

3. ***Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are : strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed in chapter 2.

4. *Form and size of the parts.* The form and size are based on judgement. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. *Frictional resistance and lubrication.* There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. *Convenient and economical features.* In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take-up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible.

The economical operation of a machine which is to be used for production, or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. *Use of standard parts.* The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order.

The standard or stock parts should be used whenever possible ; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

Design considerations play important role in the successful production of machines.

8. *Safety of operation.* Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. *Workshop facilities.* A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. *Number of machines to be manufactured.* The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of hand made samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions, should be to reduce the manufacturing cost to the minimum.

Car assembly line.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

1.4 General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows :

1. ***Recognition of need.*** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis (Mechanisms).*** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. ***Analysis of forces.*** Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection.*** Select the material best suited for each member of the machine.

5. ***Design of elements (Size and Stresses).*** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. ***Modification.*** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

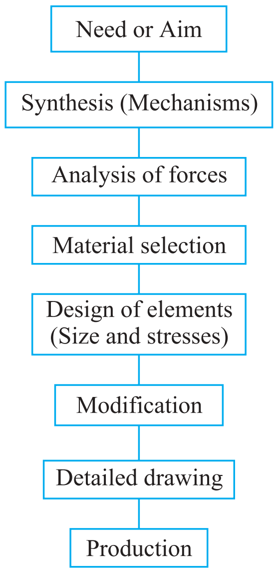


Fig. 1.1. General procedure in Machine Design.

7. ***Detailed drawing.*** Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. ***Production.*** The component, as per the drawing, is manufactured in the workshop.

The flow chart for the general procedure in machine design is shown in Fig. 1.1.

Note : When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are very important features which gives grace and lustre to product and dominates the market.

1.5 Fundamental Units

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called ***fundamental units.***

1.6 Derived Units

Some units are expressed in terms of other units, which are derived from fundamental units, are known as ***derived units*** *e.g.* the unit of area, velocity, acceleration, pressure, etc.

1.7 System of Units

There are only four systems of units, which are commonly used and universally recognised. These are known as :

1. C.G.S. units, 2. F.P.S. units, 3. M.K.S. units, and 4. S.I. units.

Since the present course of studies are conducted in S.I. system of units, therefore, we shall discuss this system of unit only.

1.8 S.I. Units (International System of Units)

The 11th General Conference* of Weights and Measures have recommended a unified and systematically constituted system of fundamental and derived units for international use. This system is now being used in many countries. In India, the standards of Weights and Measures Act 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.

In this system of units, there are seven fundamental units and two supplementary units, which cover the entire field of science and engineering. These units are shown in Table 1.1

Table 1.1. Fundamental and supplementary units.

S.No.	*Physical quantity*	*Unit*
	Fundamental units	
1.	Length (*l*)	Metre (m)
2.	Mass (*m*)	Kilogram (kg)
3.	Time (*t*)	Second (s)
4.	Temperature (*T*)	Kelvin (K)
5.	Electric current (*I*)	Ampere (A)
6.	Luminous intensity(*Iv*)	Candela (cd)
7.	Amount of substance (*n*)	Mole (mol)
	Supplementary units	
1.	Plane angle (α, β, θ, ϕ)	Radian (rad)
2.	Solid angle (Ω)	Steradian (sr)

* It is known as General Conference of Weights and Measures (G.C.W.M). It is an international organisation of which most of the advanced and developing countries (including India) are members. The conference has been entrusted with the task of prescribing definitions for various units of weights and measures, which are the very basics of science and technology today.

The derived units, which will be commonly used in this book, are given in Table 1.2.

Table 1.2. Derived units.

S.No.	Quantity	Symbol	Units
1.	Linear velocity	*V*	m/s
2.	Linear acceleration	*a*	m/s^2
3.	Angular velocity	ω	rad/s
4.	Angular acceleration	α	rad/s^2
5.	Mass density	ρ	kg/m^3
6.	Force, Weight	*F, W*	N ; 1N = $1kg\text{-}m/s^2$
7.	Pressure	*P*	N/m^2
8.	Work, Energy, Enthalpy	*W, E, H*	J ; 1J = 1N-m
9.	Power	*P*	W ; 1W = 1J/s
10.	Absolute or dynamic viscosity	μ	$N\text{-}s/m^2$
11.	Kinematic viscosity	*v*	m^2/s
12.	Frequency	*f*	Hz ; 1Hz = 1cycle/s
13.	Gas constant	*R*	J/kg K
14.	Thermal conductance	*h*	W/m^2 K
15.	Thermal conductivity	*k*	W/m K
16.	Specific heat	*c*	J/kg K
17.	Molar mass or Molecular mass	*M*	kg/mol

1.9 Metre

The metre is defined as the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels 2 p_{10} and 5 d_5 of the Krypton– 86 atom.

1.10 Kilogram

The kilogram is defined as the mass of international prototype (standard block of platinum-iridium alloy) of the kilogram, kept at the International Bureau of Weights and Measures at Sevres near Paris.

1.11 Second

The second is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium – 133 atom.

1.12 Presentation of Units and their Values

The frequent changes in the present day life are facilitated by an international body known as International Standard Organisation (ISO) which makes recommendations regarding international standard procedures. The implementation of lSO recommendations, in a country, is assisted by its organisation appointed for the purpose. In India, Bureau of Indian Standards (BIS), has been created for this purpose. We have already discussed that the fundamental units in S.I. units for length, mass and time is metre, kilogram and second respectively. But in actual practice, it is not necessary to express all lengths in metres, all masses in kilograms and all times in seconds. We shall, sometimes, use the convenient units, which are multiples or divisions of our basic units in tens. As a typical example, although the metre is the unit of length, yet a smaller length of one-thousandth of a metre proves to be more convenient unit, especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in the basic units to indicate the multiplier. The full list of these prefixes is given in the following table :

Table 1.3. Prefixes used in basic units.

Factor by which the unit is multiplied	*Standard form*	*Prefix*	*Abbreviation*
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1000	10^{3}	kilo	K
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

1.13 Rules for S.I. Units

The eleventh General Conference of Weights and Measures recommended only the fundamental and derived units of S.I. units. But it did not elaborate the rules for the usage of the units. Later on many scientists and engineers held a number of meetings for the style and usage of S.I. units. Some of the decisions of the meeting are :

1. For numbers having five or more digits, the digits should be placed in groups of three separated by spaces (instead of commas)** counting both to the left and right of the decimal point.
2. In a four*** digit number, the space is not required unless the four digit number is used in a column of numbers with five or more digits.
3. A dash is to be used to separate units that are multiplied together. For example, newton × metre is written as N-m. It should not be confused with mN, which stands for milli newton.
4. Plurals are never used with symbols. For example, metre or metres are written as m.
5. All symbols are written in small letters except the symbol derived from the proper names. For example, N for newton and W for watt.
6. The units with names of the scientists should not start with capital letter when written in full. For example, 90 newton and not 90 Newton.

At the time of writing this book, the authors sought the advice of various international authorities, regarding the use of units and their values. Keeping in view the international reputation of the authors, as well as international popularity of their books, it was decided to present **** units and

* These prefixes are generally becoming obsolete, probably due to possible confusion. Moreover it is becoming a conventional practice to use only those power of ten which conform to 10^{3x}, where x is a positive or negative whole number.

** In certain countries, comma is still used as the decimal mark

*** In certain countries, a space is used even in a four digit number.

**** In some of the question papers of the universities and other examining bodies standard values are not used. The authors have tried to avoid such questions in the text of the book. However, at certain places the questions with sub-standard values have to be included, keeping in view the merits of the question from the reader's angle.

their values as per recommendations of ISO and BIS. It was decided to use :

4500	not	4 500	or	4,500
75 890 000	not	75890000	or	7,58,90,000
0.012 55	not	0.01255	or	.01255
30×10^6	not	3,00,00,000	or	3×10^7

The above mentioned figures are meant for numerical values only. Now let us discuss about the units. We know that the fundamental units in S.I. system of units for length, mass and time are metre, kilogram and second respectively. While expressing these quantities, we find it time consuming to write the units such as metres, kilograms and seconds, in full, every time we use them. As a result of this, we find it quite convenient to use some standard abbreviations :

We shall use :

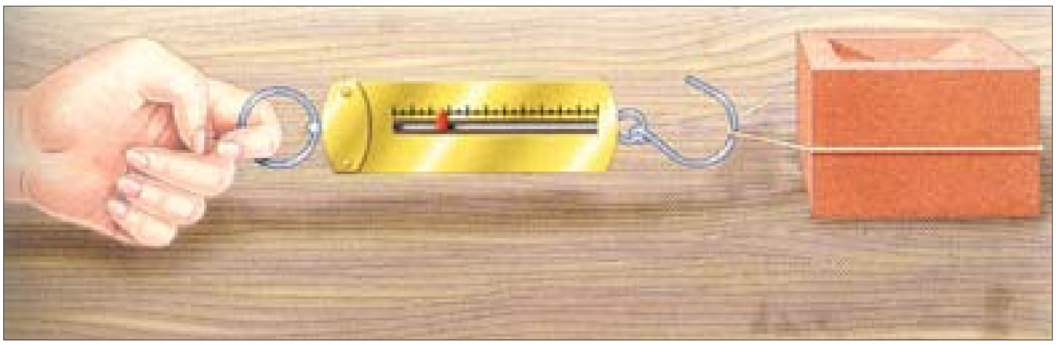
m	for metre or metres
km	for kilometre or kilometres
kg	for kilogram or kilograms
t	for tonne or tonnes
s	for second or seconds
min	for minute or minutes
N-m	for netwon × metres (*e.g.* work done)
kN-m	for kilonewton × metres
rev	for revolution or revolutions
rad	for radian or radians

1.14 Mass and Weight

Sometimes much confusion and misunderstanding is created, while using the various systems of units in the measurements of force and mass. This happens because of the lack of clear understanding of the difference between the mass and weight. The following definitions of mass and weight should be clearly understood :

Mass. It is the amount of matter contained in a given body and does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance.

Weight. It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with the distance of the body from the centre of the earth, therefore, the weight of the body will vary with its position on the earth's surface (say latitude and elevation). It is thus obvious, that the weight is a force.



The pointer of this spring gauge shows the tension in the hook as the brick is pulled along.

The earth's pull in metric units at sea level and 45° latitude has been adopted as one force unit and named as one kilogram of force. Thus, it is a definite amount of force. But, unfortunately, has the same name as the unit of mass.

The weight of a body is measured by the use of a spring balance, which indicates the varying tension in the spring as the body is moved from place to place.

Note : The confusion in the units of mass and weight is eliminated to a great extent, in S.I units . In this system, the mass is taken in kg and the weight in newtons. The relation between mass (m) and weight (W) of a body is

$$W = m.g \quad \text{or} \quad m = W / g$$

where W is in newtons, m in kg and g is the acceleration due to gravity in m/s^2.

1.15 Inertia

It is that property of a matter, by virtue of which a body cannot move of itself nor change the motion imparted to it.

1.16 Laws of Motion

Newton has formulated three laws of motion, which are the basic postulates or assumptions on which the whole system of dynamics is based. Like other scientific laws, these are also justified as the results, so obtained, agree with the actual observations. Following are the three laws of motion :

1. ***Newton's First Law of Motion.*** It states, "*Every body continues in its state of rest or of uniform motion in a straight line, unless acted upon by some external force*". This is also known as *Law of Inertia.*

2. ***Newton's Second Law of Motion.*** It states, "*The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts*".

3. ***Newton's Third Law of Motion.*** It states, "*To every action, there is always an equal and opposite reaction*".

1.17 Force

It is an important factor in the field of Engineering science, which may be defined as **an agent, which produces or tends to produce, destroy or tends to destroy motion.**

According to Newton's Second Law of Motion, the applied force or impressed force is directly proportional to the rate of change of momentum. We know that

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

Let m = Mass of the body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration, and

t = Time required to change velocity from u to v.

$\therefore$ Change of momentum $= mv - mu$

and rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = m.a \qquad \ldots \left(\because \frac{v-u}{t} = a\right)$$

or Force, $F \propto ma$ or $F = k\,m\,a$

where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces a unit acceleration to a body of unit mass.

$$\therefore \quad F = m.a = \text{Mass} \times \text{Acceleration}$$

In S.I. system of units, the unit of force is called **newton** (briefly written as N). A ***newton may be defined as the force, while acting upon a mass of one kg, produces an acceleration of 1 m/s² in the direction in which it acts.*** Thus

$$1\text{N} = 1\text{kg} \times 1\text{ m/s}^2 = 1\text{kg-m/s}^2$$

Far away from Earth's gravity and its frictional forces, a spacecraft shows Newton's three laws of motion at work.

1.18 Absolute and Gravitational Units of Force

We have already discussed, that when a body of mass 1 kg is moving with an acceleration of 1 m/s², the force acting on the body is one newton (briefly written as 1 N). Therefore, when the same body is moving with an acceleration of 9.81 m/s², the force acting on the body is 9.81N. But we denote 1 kg mass, attracted towards the earth with an acceleration of 9.81 m/s² as 1 kilogram force (briefly written as kgf) or 1 kilogram weight (briefly written as kg-wt). It is thus obvious that

$$1\text{kgf} = 1\text{kg} \times 9.81\text{ m/s}^2 = 9.81\text{ kg-m/s}^2 = 9.81\text{ N} \quad \ldots (\because 1\text{N} = 1\text{kg-m/s}^2)$$

The above unit of force *i.e.* kilogram force (kgf) is called ***gravitational*** or ***engineer's unit of force,*** whereas netwon is the ***absolute*** or ***scientific*** or ***S.I. unit of force.*** It is thus obvious, that the gravitational units are '*g*' times the unit of force in the absolute or S. I. units.

It will be interesting to know that ***the mass of a body in absolute units is numerically equal to the weight of the same body in gravitational units.***

For example, consider a body whose mass, m = 100 kg.

∴ The force, with which it will be attracted towards the centre of the earth,

$$F = m.a = m.g = 100 \times 9.81 = 981\text{ N}$$

Now, as per definition, we know that the weight of a body is the force, by which it is attracted towards the centre of the earth.

∴ Weight of the body,

$$W = 981\text{ N} = \frac{981}{9.81} = 100\text{ kgf} \quad \ldots (\because 1\text{ kgf} = 9.81\text{ N})$$

In brief, the weight of a body of mass m kg at a place where gravitational acceleration is 'g' m/s² is $m.g$ newtons.

1.19 Moment of Force

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required, and the line of action of the force. Mathematically,

$$\text{Moment of a force} = F \times l$$

where F = Force acting on the body, and

l = Perpendicular distance of the point and the line of action of the force (F) as shown in Fig. 1.2.

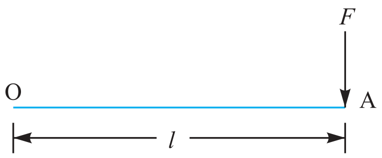


Fig. 1.2. Moment of a force.

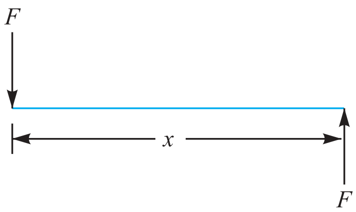


Fig. 1.3. Couple.

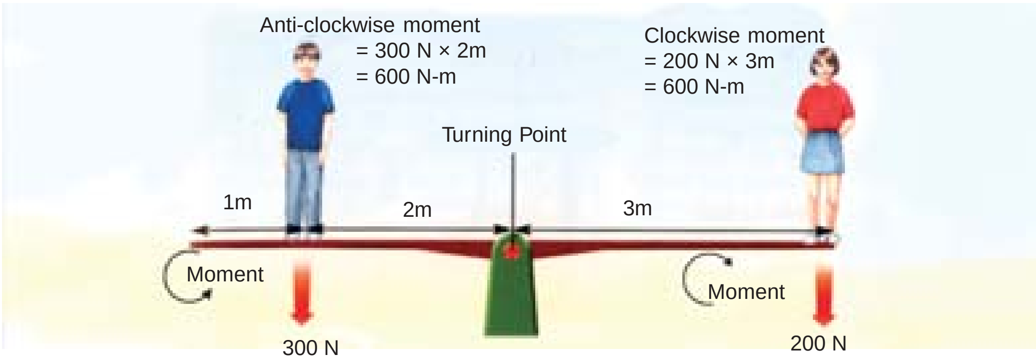
1.20 Couple

The two equal and opposite parallel forces, whose lines of action are different form a couple, as shown in Fig. 1.3.

The perpendicular distance (x) between the lines of action of two equal and opposite parallel forces is known as ***arm of the couple.*** The magnitude of the couple (*i.e.* moment of a couple) is the product of one of the forces and the arm of the couple. Mathematically,

$$\text{Moment of a couple} = F \times x$$

A little consideration will show, that a couple does not produce any translatory motion (*i.e.* motion in a straight line). But, a couple produces a motion of rotation of the body on which it acts.



A see saw is balanced when the clockwise moment equals the anti-clockwise moment. The boy's weight is 300 newtons (300 N) and he stands 2 metres (2 m) from the pivot. He causes the anti-clockwise moment of 600 newton-metres (N-m). The girl is lighter (200 N) but she stands further from the pivot (3m). She causes a clockwise moment of 600 N-m, so the seesaw is balanced.

1.21 Mass Density

The mass density of the material is the mass per unit volume. The following table shows the mass densities of some common materials used in practice.

Table 1.4. Mass density of commonly used materials.

Material	*Mass density (kg/m^3)*	*Material*	*Mass density (kg/m^3)*
Cast iron	7250	Zinc	7200
Wrought iron	7780	Lead	11 400
Steel	7850	Tin	7400
Brass	8450	Aluminium	2700
Copper	8900	Nickel	8900
Cobalt	8850	Monel metal	8600
Bronze	8730	Molybdenum	10 200
Tungsten	19 300	Vanadium	6000

1.22 Mass Moment of Inertia

It has been established since long that a rigid body is composed of small particles. If the mass of every particle of a body is multiplied by the square of its perpendicular distance from a fixed line, then the sum of these quantities (for the whole body) is known as ***mass moment of inertia*** of the body. It is denoted by *I*.

Consider a body of total mass *m*. Let it be composed of small particles of masses m_1, m_2, m_3, m_4, etc. If k_1, k_2, k_3, k_4, etc., are the distances from a fixed line, as shown in Fig. 1.4, then the mass moment of inertia of the whole body is given by

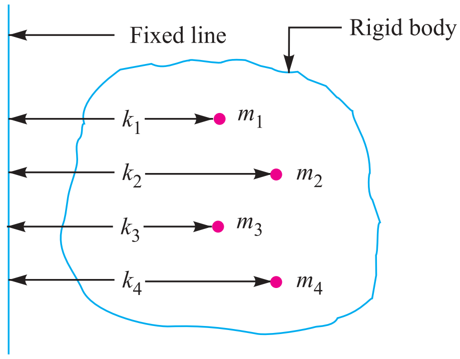


Fig. 1.4. Mass moment of inertia.

$$I = m_1 (k_1)^2 + m_2 (k_2)^2 + m_3 (k_3)^2 + m_4 (k_4)^2 + ...$$

If the total mass of a body may be assumed to concentrate at one point (known as centre of mass or centre of gravity), at a distance *k* from the given axis, such that

$$mk^2 = m_1 (k_1)^2 + m_2 (k_2)^2 + m_3 (k_3)^2 + m_4 (k_4)^2 + ...$$

then $$I = m\,k^2$$

The distance *k* is called the ***radius of gyration.*** It may be defined ***as the distance, from a given reference, where the whole mass of body is assumed to be concentrated to give the same value of I.***

The unit of mass moment of inertia in S.I. units is kg-m^2.

Notes : 1. If the moment of inertia of body about an axis through its centre of gravity is known, then the moment of inertia about any other parallel axis may be obtained by using a parallel axis theorem *i.e.* moment of inertia about a parallel axis,

$$I_p = I_G + mh^2$$

where I_G = Moment of inertia of a body about an axis through its centre of gravity, and

h = Distance between two parallel axes.

2. The following are the values of *I* for simple cases :

(*a*) The moment of inertia of a thin disc of radius *r*, about an axis through its centre of gravity and perpendicular to the plane of the disc is,

$$I = mr^2/2 = 0.5\,mr^2$$

and moment of inertia about a diameter,

$$I = mr^2/4 = 0.25\,mr^2$$

(*b*) The moment of inertia of a thin rod of length *l*, about an axis through its centre of gravity and perpendicular to its length,

$$I_G = ml^2/12$$

and moment of inertia about a parallel axis through one end of a rod,

$$I_p = ml^2/3$$

3. The moment of inertia of a solid cylinder of radius *r* and length *l*,about the longitudinal axis or polar axis

$$= mr^2/2 = 0.5\,mr^2$$

and moment of inertia through its centre perpendicular to the longitudinal axis

$$= m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$$

1.23 Angular Momentum

It is the product of the mass moment of inertia and the angular velocity of the body. Mathematically,

$$\text{Angular momentum} = I.\omega$$

where I = Mass moment of inertia, and

ω = Angular velocity of the body.

1.24 Torque

It may be defined as the product of force and the perpendicular distance of its line of action from the given point or axis. A little consideration will show that the torque is equivalent to a couple acting upon a body.

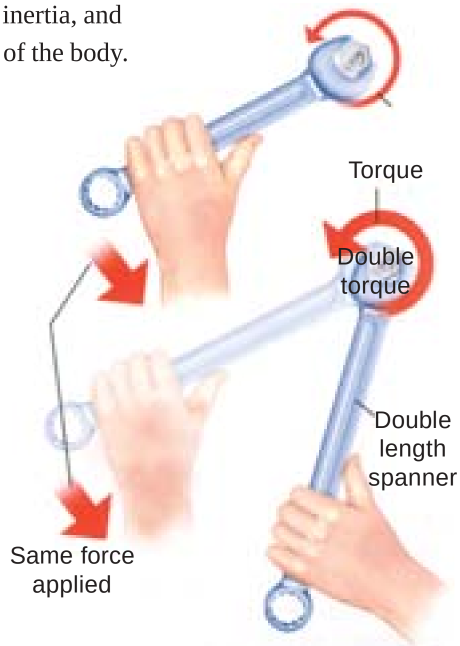
The Newton's second law of motion when applied to rotating bodies states, the ***torque is directly proportional to the rate of change of angular momentum.*** Mathematically,

$$\text{Torque, } T \propto \frac{d(I\omega)}{dt}$$

Since I is constant, therefore,

$$T = I \times \frac{d\omega}{dt} = I.\alpha$$

$$\ldots \left[\because \frac{d\omega}{dt} = \text{Angular acceleration } (\alpha)\right]$$



Same force applied at double the length, doubles the torque.

1.25 Work

Whenever a force acts on a body and the body undergoes a displacement in the direction of the force, then work is said to be done. For example, if a force F acting on a body causes a displacement x of the body in the direction of the force, then

$$\text{Work done} = \text{Force} \times \text{Displacement} = F \times x$$

If the force varies linearly from zero to a maximum value of F, then

$$\text{Work done} = \frac{0 + F}{2} \times x = \frac{F}{2} \times x$$

When a couple or torque (T) acting on a body causes the angular displacement (θ) about an axis perpendicular to the plane of the couple, then

$$\text{Work done} = \text{Torque} \times \text{Angular displacement} = T.\theta$$

The unit of work depends upon the units of force and displacement. In S. I. system of units, the practical unit of work is N-m. It is the work done by a force of 1 newton, when it displaces a body through 1 metre. The work of 1 N-m is known as joule (briefly written as J), such that 1 N-m = 1 J.

Note : While writing the unit of work, it is a general practice to put the units of force first followed by the units of displacement (*e.g.* N-m).

1.26 Power

It may be defined as the rate of doing work or work done per unit time. Mathematically,

$$\text{Power, } P = \frac{\text{Work done}}{\text{Time taken}}$$

In S.I system of units, the unit of power is watt (briefly written as W) which is equal to 1 J/s or 1N-m/s. Thus, the power developed by a force of F (in newtons) moving with a velocity v m/s is $F.v$ watt. Generally, a bigger unit of power called kilowatt (briefly written as kW) is used which is equal to 1000 W

Notes : 1. If T is the torque transmitted in N-m or J and ω is angular speed in rad/s, then

$$\text{Power, } P = T.\omega = T \times 2\pi N / 60 \text{ watts} \qquad \ldots (\because \omega = 2\pi N/60)$$

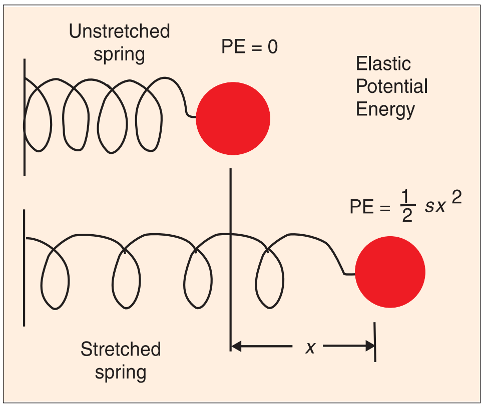
where N is the speed in r.p.m.

2. The ratio of the power output to power input is known as ***efficiency*** of a machine. It is always less than unity and is represented as percentage. It is denoted by a Greek letter eta (η). Mathematically,

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}}$$

1.27 Energy

It may be defined as the capacity to do work. The energy exists in many forms *e.g.* mechanical, electrical, chemical, heat, light, etc. But we are mainly concerned with mechanical energy.



The mechanical energy is equal to the work done on a body in altering either its position or its velocity. The following three types of mechanical energies are important from the subject point of view :

1. *Potential energy.* It is the energy possessed by a body, for doing work, by virtue of its position. For example, a body raised to some height above the ground level possesses potential energy, because it can do some work by falling on earth's surface.

Let W = Weight of the body,

m = Mass of the body, and

h = Distance through which the body falls.

∴ Potential energy,

$$\text{P.E.} = W.\,h = m.g.h$$

It may be noted that

(*a*) When W is in newtons and h in metres, then potential energy will be in N-m.

(*b*) When m is in kg and h in metres, then the potential energy will also be in N-m as discussed below :

We know that potential energy

$$= m.g.h = \text{kg} \times \frac{\text{m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \ldots \left(\because 1\text{N} = \frac{1\ \text{kg-m}}{\text{s}^2}\right)$$

2. *Strain energy.* It is the potential energy stored by an elastic body when deformed. A compressed spring possesses this type of energy, because it can do some work in recovering its original shape. Thus, if a compressed spring of stiffness (s) N per unit deformation (*i.e.* extension or compression) is deformed through a distance x by a weight W, then

$$\text{Strain energy} = \text{Work done} = \frac{1}{2} W.x = \frac{1}{2} s.x^2 \qquad \ldots (\because W = s.x)$$

In case of a torsional spring of stiffness (q) N-m per unit angular deformation when twisted through an angle θ radians, then

$$\text{Strain energy} = \text{Work done} = \frac{1}{2}\, q.\theta^2$$

3. ***Kinetic energy.*** It is the energy possessed by a body, for doing work, by virtue of its mass and velocity of motion. If a body of mass m attains a velocity v from rest in time t, under the influence of a force F and moves a distance s, then

$$\text{Work done} = F.s = m.a.s \qquad ...(\because F = m.a)$$

∴ Kinetic energy of the body or the kinetic energy of translation,

$$\text{K.E.} = m.a.s = m \times a \times {}^{*}\frac{v^2}{2a} = \frac{1}{2}\, mv^2$$

It may be noted that when m is in kg and v in m/s, then kinetic energy will be in N-m as discussed below :

We know that kinetic energy,

$$\text{K.E.} = \frac{1}{2}\, mv^2 = \text{kg} \times \frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg - m}}{\text{s}^2} \times \text{m} = \text{N-m} \quad ...\left(\because 1\text{N} = \frac{1\ \text{kg-m}}{\text{s}^2}\right)$$

Notes : 1. When a body of mass moment of inertia I (about a given axis) is rotated about that axis, with an angular velocity ω, then it possesses some kinetic energy. In this case,

$$\text{Kinetic energy of rotation} = \frac{1}{2}\, I.\omega^2$$

2. When a body has both linear and angular motions, *e.g.* wheels of a moving car, then the total kinetic energy of the body is equal to the sum of linear and angular kinetic energies.

$$\therefore \qquad \text{Total kinetic energy} = \frac{1}{2}\, m.v^2 + \frac{1}{2}\, I.\omega^2$$

3. The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which energy can exist. This statement is known as ***'Law of Conservation of Energy'***.

4. The loss of energy in any one form is always accompanied by an equivalent increase in another form. When work is done on a rigid body, the work is converted into kinetic or potential energy or is used in overcoming friction. If the body is elastic, some of the work will also be stored as strain energy.

---

* We know that $v^2 - u^2 = 2\, a.s$
Since the body starts from rest (*i.e.* $u = 0$), therefore,
$v^2 = 2\, a.s$ or $s = v^2 / 2a$

GO To FIRST